# Designing with Functional Representations: GUI and Solver 

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## Formats for Fabrication

■ How do we represent objects?

- 2D areas and 3D volumes
- Design $\rightarrow$ fabrication


## Boundary Representations

Data describing surface of an object
User Fersp



## Boundary Representations

Advantages:
■ Easy to render
■ Long history

- Common in computer graphics


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Advantages:
■ Easy to render
■ Long history

- Common in computer graphics

Disadvantages:

- Finite resolution
- Requires surface $\rightarrow$ volume conversion

■ Constructive solid geometry is hard / messy

## Boundary Representations



## Functional Representation



$$
\mathrm{X} * \mathrm{X}+\mathrm{Y} * \mathrm{Y}<1
$$

## Functional Representation



## Functional Representation

- Resolution-independent
- Platform-independent

■ Easy to transform and modify
■ Hard to render

## Design Tools

- Library of common shapes and operators - Python scripts as design files
- Interactive GUI:



## Solver Fundamentals

How to convert an expression into an image?

$$
(X * X+Y * Y<1) \& \&(X * X+Y * Y>0.5)
$$



## Solver Fundamentals

■ Previous solver:

- Brute-force evaluation
- Paste expression into template $C$ program
- Compile \& run!

■ Evaluates expression for every pixel

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■ Previous solver:

- Brute-force evaluation
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■ Evaluates expression for every pixel

- We can do better.


## Solver Architecture

■ Parser
■ Converts string into tree structure

- Optimizes tree structure

■ Solver
■ Evaluates expression on region
■ Interval arithmetic speeds up evaluation
■ Uses caching and multithreading

## Parser

Expressions $\rightarrow$ trees

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## $\mathrm{X}+\mathrm{Y}>0$ becomes



## Parser

## Expressions $\rightarrow$ trees

## $\mathrm{X}+\mathrm{Y}>0$ becomes



Uses shunting-yard algorithm

## Tree Structure

Tree of expressions operating on constants
variables other expressions


## Tree Structure

Distinct data types:
Floating-point value/interval
Tri-bool (true, false, or ambiguous)
Color (32-bit integer)


## Architecture

- Parser

■ Converts string into tree structure

- Optimizes tree structure


## - Solver

■ Evaluates expression on region

- Interval arithmetic speeds up evaluation
- Uses caching and multithreading


## Interval Arithmetic

■ Operations are applied to regions in space

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■ Operations are applied to regions in space

- Logic operations are true, false, or ambiguous
- $[-1,1]<2$ is true
- $[-1,1]<-2$ is false
- $[-1,1]<0$ is ambiguous


## Subdivision \& Recursion

Solver algorithm:

- Evaluate on initial region
- If true or false, color and return
- If ambiguous, subdivide and recurse


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Solver algorithm:

- Evaluate on initial region
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Regions below a minimum size are evaluated point-by-point, which improves performance.

## Subdivision \& Recursion



## Performance



## Future Work

■ Improving GUI design tools

- Generating surfaces
- Improving standard library

■ Possibly switching to GPU

## Resources



## Questions?

```
1 from cad_shapes import
3 DEPTH = 15
4
6 b = 'Y'
8 mbrot - '0'
g for i in range(DEPTH):
10 prev_level = '(pow(%s,2) + pow(%s,2))<4'% (a,b)
11 (a, b) = ('(pow(%s, 2)-pow(%s, 2)+X)' % (a, b),
    (a, b) = ('(pow(%s, 2)-pow(%s, 2)+X)'
        this_level = '(pow(%s,Z) + pow(%s,2)) < 4' % (a,b)
    this_level = subtract(prev_level, this_level)
    value = (int(pow(float(i+1)/DEPTH, 3) * 200.0) << 0) + )
                (int(pow(float(i+1)/DEPTH, 2) * 200.0) << 8) + \
                    (int(pow(float(i+1) / DEPTH, 1) * 200.0) << 16)
    mbrot = odd(mbrot, color(value, this_level))
18
20 mbrot = add(mbrot, color(white, '(pow(%s,z) + pow(%s,Z)) < 4' % (a,b)))
#1 * Render boundaries
z2 cad.xmin = -2
23 cad. xmax =0.7
24 cad.ymin = -1.2
25 cad.ymax = 1.2
26 cad.mm_per_unit = 25.4 # inch units
27 cad.type = "RGB" # Boolean or RGB
28
29 cad.function = mbrot
```



## Extra Slides

Parser-Level Optimizations

## Tree Simplification


$(\mathrm{X}+0) *(\mathrm{Y}+0)<1$

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$(\mathrm{X}+0) *(\mathrm{Y}+0)<1$

## Node Combination


$(\mathrm{X}+1) *(\mathrm{X}+1)+\mathrm{Y} * \mathrm{Y}<1$

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$(\mathrm{X}+1) *(\mathrm{X}+1)+\mathrm{Y} * \mathrm{Y}<1$

## Extra Slides

## Solver-Level Optimizations

## Branch Caching


$(X>0) \& \&(X * X+Y * Y<1)$

## Branch Caching



## Branch Caching



## Branch Caching



## Multithreading

■ Problem has parallel structure

- Distribute work over multiple cores:
- Divide region evenly
- Assign each core a subregion
- GPU could also be used


## Z-culling

■ For 3D objects, goal is height-map
■ Skip evaluation if region is occluded


## Extra Slides

Test Procedures \& Results

## Test Files



Alien

## Test Files



Bearing

## Test Files



Castle

## Test Files



Gear

## Test Files



PCB

## File Statistics

|  | Dimensions |  |  | Volume | File size |
| :---: | :---: | :---: | :---: | :---: | :---: |
| File | W | H | D | (MPixels) | (chars) |
| alien | 3555 | 3555 | 1 | 12.6 | 1880 |
| bearing | 711 | 711 | 237 | 119.8 | 1414 |
| castle | 447 | 447 | 203 | 40.6 | 49854 |
| gear | 1904 | 1904 | 1 | 3.6 | 8128 |
| pcb | 2273 | 1460 | 1 | 3.3 | 378743 |

## Speed Test Procedure

■ Enable/disable one optimization (with all others optimizations disabled/enabled)

- Run 10x

■ Find average run time
■ Calculate speedup/slowdown
Caveat:
Behavior is sensitive to the selected resolution

## Results



## Results



## Extra Slides

## Implementation \& Code Details

## Implementation Details

- 4,370 lines of $C++$.
- Inheritance is used for Node classes
- Parent class Node is derived into
- NonaryNode
- UnaryNode
- BinaryNode
(which are further derived into operator classes)


## Evaluation Procedure

- Two solve functions:

■ Float (single point)

- Interval (region)

■ Nodes store results of evaluation locally
■ Nodes with children look up children's locally stored results
■ Children must be evaluated before parents!

## Tree Data Structure

■ Lists of nodes, sorted by weight into levels

- Variables and constants: weight $=0$
- Others: weight $=$ max(child weights) +1

■ Evaluate nodes with weight $=0$, then nodes with weight $=1$, then nodes with weight $=$ 2 , etc.

- This order of evaluation ensures that children are evaluated before parents.


## Branch Cache Implementation

■ Each level keeps a count of "active nodes"
■ "Push" (recursing on sub-interval):

- Swap unambiguous nodes to the back of the list
- Deactivate children of unambiguous nodes
- Decrement active node count.
- Save the number of cached nodes

■ "Pop" (returning from recursion):

- Increment active node count
- Revive cached nodes
- Activate children of revived nodes

